Př. 4

$\lambda = 12$ $\quad$ $m = 4$

$\mu = 5$ $\quad$ $m - n = 2$

$n = 2$ $\quad$ $P_{odm} = P_m = P_4$

~~Jaká je P-st, že ve frontě jsou alespoň 2 zák.~~

$\cancel{P = 1 - (P_0 +}$

1. P_{odm}
2. ES, EK, EL

$\boxed{1.}$ $\rho = \frac{\lambda}{n \cdot \mu} = \frac{12}{2 \cdot 5} = \frac{12}{10} = 1{,}2$

$$P_0 = \frac{1}{\sum_{k=0}^{n} \frac{1}{k!} \left(\frac{\lambda}{\mu}\right)^k + \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n \cdot \rho \cdot \frac{1 - \rho^{m-n}}{1 - \rho}}$$

$$P_0 = \frac{1}{\frac{1}{0!} \cdot \left(\frac{12}{5}\right)^0 + \frac{1}{1!}\left(\frac{12}{5}\right)^1 + \frac{1}{2!}\left(\frac{12}{5}\right)^2 + \frac{1}{2!}\left(\frac{12}{5}\right)^2 1{,}2 \cdot \frac{1 - 1{,}2^2}{1 - 1{,}2}} =$$

$$P_0 = \frac{1}{(1 + 2{,}4 + 2{,}88) + \left(2{,}88 \cdot 1{,}2 \cdot \frac{-0{,}44}{-0{,}2}\right)} = \frac{1}{6{,}28 + 7{,}60} = 0{,}07$$

$P_0 = 7{,}2\,\%$

$$P_{odm} = P_m = \frac{1}{n!\, n^{m-n}} \cdot \left(\frac{\lambda}{\mu}\right)^m \cdot P_0$$

$$P_{odm} = P_m = P_4 = \frac{1}{2!\, 2^2} \cdot \left(\frac{12}{5}\right)^4 \cdot 0{,}072 = 0{,}298 = \underline{\underline{29{,}8\%}}$$

$\boxed{2.}$

$$ES = \frac{\lambda}{\mu} \cdot (1 - P_m) = \frac{12}{5} \cdot (1 - 0{,}298) = 1{,}698$$

$$EL = \sum_{l=1}^{m-n} l \cdot P_{n+l} = 1 \cdot P_3 + 2 \cdot P_4 = 1 \cdot 0{,}248 + 2 \cdot 0{,}298$$

$$P_3 = \frac{1}{2!\, 2^1} \cdot \left(\frac{12}{5}\right)^3 \cdot 0{,}072 = 0{,}248$$

$$EL = 0{,}844$$

$$EK = ES + EL = 2{,}542$$

$\boxed{Př. 5}$

$\lambda = 60$

$\mu = 40$

$n = 2$

$P_{odm} = 0$

1. P-st, že ve frontě jsou alespoň 2 zákazníci

$$P_{2+} = 1 - (P_0 + P_1 + P_2 + P_3)$$

2. P-st, že příchozí zák nemusí čekat

$$P = P_0 + P_1$$

$$\rho = \frac{\lambda}{n \cdot \mu} = \frac{60}{2 \cdot 40} = 0{,}75$$

$$\rho = 0{,}75$$

$$P_0 = \frac{1}{\frac{1}{0!}\cdot\left(\frac{60}{40}\right)^0 + \frac{1}{1!}\left(\frac{60}{40}\right)^1 + \frac{1}{2!}\left(\frac{60}{40}\right)^2 + \frac{1}{2!}\left(\frac{60}{40}\right)^2 \cdot \frac{0{,}75}{1-0{,}75}} =$$

$$P_0 = \frac{1}{(1 + 1{,}5 + 1{,}125) + (1{,}125 \cdot 3)} = \frac{1}{7} = 0{,}143$$

$$P_0 = 0{,}143$$

$$P_1 = \frac{1}{1!}\left(\frac{60}{40}\right)^1 \cdot 0{,}143 = 0{,}2145$$

$$P_2 = \frac{1}{2!}\left(\frac{60}{40}\right)^2 \cdot 0{,}143 = 0{,}161$$

$$P_3 = \frac{1}{2!\,2^1}\left(\frac{60}{40}\right)^3 \cdot 0{,}143 = 0{,}121$$

$$P_{2+} = 1 - (P_0 + P_1 + P_2 + P_3)$$

$$P_{2+} = 1 - (0{,}143 + 0{,}2145 + 0{,}161 + 0{,}121)$$

$$\boxed{P_{2+} = 0{,}36 = 36\,\%}$$

2. $P_0 + P_1 = 0{,}143 + 0{,}2145 = 0{,}3575 = \boxed{35{,}75\,\%}$